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## MATHEMATICS IN COMMERCIAL WORK.

THERE is need, at intervals, in most general courses of study, to rediscover mathematics; to determine, again and again, in the light of the best thought and method of the time, its educative, disciplinary, and purely utilitarian value, its place in the course, its subject-matter.

With a wealth of new matter in pure and applied science, admirably developed, constantly demanding an increasing weight in overburdened curricula, mathematics has been relegated, at times, to a less important position than formerly. No doubt this is right in some cases; in others, it follows from a failure to realize fully its possibilities of development. In the larger scientific and engineering schools alone has it retained, noticeably, relative weight, because it has been seen to be the bed-rock on which, to a large degree, the superstructure of technical training must be built.

In our modern secondary schools of commerce, however, we need not to rediscover, but to discover, mathematics, in the light of the purpose of the school. Content that secondary algebra and geometry, plus utilitarian commercial arithmetic, should constitute the commercial course in mathematics, we are only beginning to realize the possibilities of correlation with other subjects, and of a distinctive selection and treatment of the subject-matter that shall be highly educative throughout, while strongly utilitarian in parts.

These subjects, commonly required, are not new, and no new ones should be introduced, although opportunities should be given in the final year for electives of a higher order. But the standard subjects should be commercialized, to a slight degree at least, where this is possible without decreasing the efficiency resulting from a realization of the full educational and disciplinary value as a natural complement to the purely utilitarian.

Furthermore, the steps of development must be natural and progressive, each giving power to master the next. Mathemat-

ics must not be presented ready-made. The individual must make his own as the race has done; not, however, as if the race had never done it. While preventing waste of energy, the amount given by authority of teacher or book must be reduced to a minimum.

That which is distinctively utilitarian in the course must be thoroughly practical and in accord with modern usage. Business arithmetic especially is undergoing marked changes in system and development. The arithmetic of today in method and application is not that of a late yesterday.

Educationally, the greater value as to results must come from those parts which appeal to the pupil's life, activities, and interests. Mathematics should be, as far as possible, live and not dead matter. The arithmetic we know is the outcome of daily needs in every phase of life; the other branches may touch life at many points. And in this close contact, interest will start and flourish. "The mainspring of mathematical ability in a race is the attempt to adjust means accurately and economically to a given end."

As a whole, the course must give power, vigor, and strength to the mind; cultivating clear thinking, and ability to see all sides of a question; developing that individual capacity which is needed in every form of mental activity. A magnificent exercise in logic, it may sacrifice, at times, the teaching of facts, if it only gives power to prove facts.

In the light of these requirements, commercial arithmetic must be abridged by cutting off obsolete subjects and complicated methods; and enriched, by increasing greatly the quantity of simple calculation, and of modern, practical, concrete problems, especially those that deal with our active participation in life. Although business grows steadily more complex, details of organization and methods of work tend steadily toward simplicity.

This tendency results in increased demands for accuracy in fundamental processes of arithmetic, and for a working knowledge of the principles of percentage, and of elementary business principles, with ability to apply them in an increasing number of

ways. The arithmetic of actual business is suggestive, when studied at close range. It discloses:

1. That common fractions are uncommon, those with denominators of two, three, four, six, and eight alone finding extended use. For others, the nearest two-place decimal is the common substitute.

2. That quantities are generally expressed in one or two denominations. The merchant sells  $1\frac{3}{4}$  yards; not 1 yard, 2 feet, 3 inches; the grocer,  $1\frac{1}{2}$  lbs.; not 1 pound, 8 ounces; the engineer measures in feet and hundreds of feet—a decimal system—not in feet and rods.

3. That the majority of numbers expressing quantity and value are exceedingly simple. It follows naturally that ability to work mentally should be cultivated, even if the volume of modern business did not demand it. Employees waste time, energy, and frequently costly stationery on unnecessary paper calculations. Yet mental calculation, once a habit, is always easier.

4. That in actual business there is little recognition of textbook case or subject. A single real estate problem may involve simple percentage, taxes, commission, insurance, interest. Solutions must rest on the bed-rock of fundamental principles, not on the shifting sands of arbitrary cases.

5. That actual problems are frequently so expressed as to make essential ability to see as well as to solve them. A book says: "I bought 40 chairs @ \$8.40, less 15 per cent discount, paying freight of \$11.20. Terms: 30 days; 2 per cent cash. I paid cash. Find the marked price to gain 15 per cent." A similar problem I overheard expressed in these words from dealer to clerk:

"John, we want to clear 15 per cent on this invoice!"—handing him a bill. And John noted terms, discounts, prices, allowed for freight and store burden, and marked his chairs. The employer says: "Do this!" The clerk must make the problem and find or select the values necessary for its solution.

6. That calculation tables for interest, discount, insurance, taxes, wages, earthwork, etc., are commonly used to save time and insure accuracy.

7. That the use of ruled forms, many requiring extensions and calculations for which text-book courses do not prepare, is increasing rapidly.

The course in arithmetic now, to meet business and educational requirements, must be woven together by mental exercises. These may average, to advantage, one-half the recitation periods. Mental calculation finds its first field in rapid reviews of fundamental processes in whole numbers and in common and decimal fractions ; its second, in exercises in numbers under 100, continued throughout the course, and in percentage and interest, all intended to develop speed, accuracy, and knowledge of number combinations. It is the tool for systematic review and for developing shorthand arithmetic. Ready-made short methods must be handled with exceeding care. There is danger that they will go off the wrong way, or at the wrong time, or — not go off at all. But those developed instinctively by the pupil, through increasing knowledge of number in combination, remain with him — a valuable business capital.

Moreover, every practical topic may be introduced and developed through brief, pointed mental problems ; and drill secured by a series of related mental problems ; a series with the same central idea ; a statement or value on the board, around which exercises may be woven, permitting the pupil to concentrate attention solely on the new points involved.

In close co-ordination with the mental work, the written exercises and test problems “ clinch ” the subject. These should be brief, practical, living questions, at times expressed in memoranda or bill form, in order that the problem may be determined as well as solved ; at times, grouped to relate to the same business, or business condition, for related problems have far more educational value than those having simply the arbitrary connection of the text-book case.

But from arithmetic principles and terms, in combination with business terms and forms, has been evolved a business language in which business transactions are expressed and business records written. Its literature consists of notes, drafts, bills, estimates, books of records ; its phraseology of symbols, business expres-

sions, terms, forms of tabulation. Some knowledge of it the pupil gains from his bookkeeping, but it is acquired more effectively through applied arithmetic, which should form the next step in the mathematical course.

Commence with sales and order sheets, requiring horizontal and vertical addition; follow with carefully graded bills of different businesses, reading and solving the problems involved, studying the meaning and relative value of "terms" and the essentials of form. One wholesale bill, with discounts and choice of terms, contains several "pages worth" of text-book problems. Master, then, commission forms; use actual notes for interest, discount, and partial payments. Solve office paper, pay-rolls, requisitions, inventories — the field of arithmetic as recorded in business paper. At every step, too, require the preparation of original paper, having it checked and audited by the class.

Finally, later in the course, when the pupil has gained strength of mind, breadth of outlook, and a knowledge of business conditions, study in detail some of the greater problems based on arithmetic. Those of banking and finance, of insurance, annuities and endowments, of taxation and duties; the use and proper design of working tables; the effective preparation of statistics, the great problems of "cost-keeping" and factory mathematics. Here, in its highest phase, arithmetic may touch and interpret the work of most other departments of the school.

The course, thus outlined in salient points, is highly utilitarian, yet, when one has taught it, he finds it just as highly educational. The pupil, made wide-awake, finds suggestive problems and illustrations, which the competent teacher directs to emphasizing the main points under discussion, in his elementary law, commercial geography and bookkeeping, and in active life.

The principle of fair settlement, which underlies so many business arithmetic processes, and other business and ethical principles, are constantly emphasized. The unusual opportunities for individual and original work bring breadth of mind and training in system, form, and arrangement; while class discussions and rigid analyses give ability to judge before solving, to reason accurately, and to do away with that inaccuracy of statement which is the parent of inaccuracy of thought.

Algebra is not taught, distinctively, for its utility to the coming merchant, although to the mathematician and to the engineer it is indispensable. Its greater value is as an exercise in applied logic, where it gives character to the teacher's work and raises it to the plane of true education. A source of mind-power, it develops capacity to master subjects of kindred, or of totally different, nature.

Algebra, in part, is distinctively universal arithmetic, and the two subjects work well in double harness. Elementary algebra and arithmetic, in combination, should precede commercial arithmetic, for the methods of algebraic reasoning aid in mastering arithmetical problems, the method of equation solving easily what is otherwise difficult. This suggests, also, the substitution of practical business arithmetic, problems for the many objectionable applied exercises in algebra, now in print.

A scientific treatment of the subject should lead from the beginning to the equation, which should be introduced early, and emphasized, until the pupil is familiar with the principles on which the processes of operation are based. Factoring and its relation to equations and fractions should also be a strong feature. In work of this class the mental exercises should develop the same accuracy and facility in handling the literal as later the numerical. In all stages of the work methods of checking solutions are important in cultivating a valuable business habit, and in encouraging independence as well.

That part of higher algebra, less distinctively universal arithmetic, covering the theories of combinations and probabilities has also sufficient value, from practical and disciplinary standpoints, to warrant its rigid treatment. On problems of life insurance and in studies of various business conditions, it will be found to have direct bearing.

Geometry claims place especially because of its value as an exercise in formal logic, although in parts, in mensuration for example, it has high utilitarian value. In class it is often effectively taught as a combination of the inventional and the demonstrative. The inventional, leading to a right conception of the truths to be established, introduces naturally the deductive

method of establishing them. Elementary ideas of logic, however, may be introduced from the beginning, and demonstrations made exceedingly rigid—with the rigor consisting in soundness of structural development, as well as in clearness and effectiveness of expression.

The field of demonstrative work should include plane geometry and the principal theorems of solid, many of the latter having unusual disciplinary value.

The field of applied work should be as broad as time allows, for here is possible correlation with other subjects, and contact with actual life. The practical problems of mensuration, the preparation of plans and estimates, designing, pattern-making, the geometrical representation of statistics, suggest lines of development.

The value of geometry is measured, to an unusual degree, in terms of the teacher. His insistence on rigid demonstration and clear statement, especially in oral work, and on neat, accurate, effective figures; his method of expression before the class; his choice of original exercises for assignment at every stage of the work; his methods of review, measure the value of geometry to discipline the mind, to arouse interest, to inculcate habits of neatness, order, diligence, honesty.

The final year should offer opportunities for advanced elective work, consisting principally of trigonometry, or of problems relating to heat, light, and power, with which many business men need familiarity, but preferably of descriptive geometry. From experience with classes in this latter subject, I regard it as the most attractive subject-matter mathematics has to offer. As a theoretical subject it has no mathematical equal in arousing general class interest, while it develops a high degree of mind-power. Its applications, in practice, also, cover an exceedingly broad and interesting field. The Committee of Ten, speaking of projective geometry, which includes descriptive, says:

It is astonishing that this subject should be so generally ignored, for mathematics offers nothing more attractive. It possesses the concreteness of the ancient geometry without the tedious particularity, and the power of the analytical without the reckoning, and by the beauty of its ideas and methods,



illustrates the æsthetic quality which is the charm of the higher mathematics, but which the elementary mathematics, in general, lacks.

Although little has been said directly, enough has been suggested to show the natural order of the subjects. Details of exact order, and of number of recitations, must depend on the other courses of the school, with which the mathematical work must be properly co-ordinated. It is safe to add, however, that in a time no greater than, and more probably less than, the average period devoted to the other major subjects, mathematics may claim, if properly treated, at least equal weight with these in progressively expanding the interests and powers of the pupil, in training him to think clearly, to plan, to organize, to see relations; in developing the broad active man of affairs.

But this requires the right man behind the mathematics!

"It would be a good thing," reads a rule of the Franke Institute, laid down two centuries ago for the guidance of teachers, "if the teacher would himself work through the book so that he could help the children. It would be a good thing today, if the teacher would read deeply in the living book of his subject, in the book also, of business life and activity. A broad man, he should understand the purpose and principles of the other subjects of the curriculum, drawing inspiration and illustration from them, yet be ever sharpened to a mathematical point.

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